Research Papers

PLANE WAVE SOLUTION OF FIELD EQUATIONS $R_{ij} = 0$ IN FIVE DIMENSIONAL SPACE-TIME

L. S. Ladke

Department of Marhematics ,
Sarvodaya Mahavidyalaya Sindewahi Distt. Chandrapur India.

Abstract

Plane wave solutions of field equations $R_{ij} = 0$ in V_5 are given by g_{ij} which satisfies
$g_{\mu\mu} < 0, \quad \mu = 1, 2, 3, 4, \quad g55 > 0$

$g_{ij} = g_{ij}(Z), \qquad Z = Z(u, x, y, z, t)$

$\phi_1 w^3 + \phi_2 w^4 + w^5 = 0$

$\rho_a = \overline{g}_{ai} w^i = 0, \qquad R_{a\alpha} = 0, \quad a = 1, 2, \quad \alpha = 3, 4, 5.$

and $Q\rho_{\alpha\beta} + P\sigma_{\alpha\beta} = 0$

which is equivalent to

$\overline{w}_1 \rho_{\alpha\beta} + \overline{w}_1 \sigma_{\alpha\beta} = 0 = \overline{\phi}_1 \rho_{\alpha\beta} + \overline{\phi}_1 \sigma_{\alpha\beta},$

$\overline{w}_2 \rho_{\alpha\beta} + \overline{w}_2 \sigma_{\alpha\beta} = 0 = \overline{\phi}_2 \rho_{\alpha\beta} + \overline{\phi}_2 \sigma_{\alpha\beta}$

If Z is independent of y, the work of Adhav and Karade (1994)regarding plane wave solutions in V_5 can be obtained from our investigations.

KEYWORDS:

Plane Wave Solution , Five Dimensional Space , generalization .

I.INTRODUCTION

Having defined plane wave as a non – flat solution of field equations $R_{ij} = 0$, Takeno (1961) has organized the solutions of field equations in the following format

$\rho_a = 0, \qquad R_{a\alpha} = 0, \quad a = 1, 2, \quad \alpha = 3, 4$

And $N\rho_{\alpha\beta} + M\sigma_{\alpha\beta} = 0$ (1)

Where equation (1) is equivalent to

$$\bar{\bar{w}}\,\rho_{\alpha\beta} + \bar{w}\,\sigma_{\alpha\beta} = 0 = \bar{\bar{\phi}}\,\rho_{\alpha\beta} + \bar{\phi}\,\sigma_{\alpha\beta},$$

Here $t + \phi z = w$, $\phi = \frac{Z_{,3}}{Z_{,4}}$ $M = \bar{w} - \bar{\phi} z.$ $N = \bar{\bar{w}} - \bar{\bar{\phi}} z$

and a bar (-) over the letter denotes the derivative with respect to Z. Extension of Takeno's (1961) work in v_4 to higher five dimension was further carried out by Adhav and Karade (1994) . They have reformulated the definition of plane wave as follows :

Definition A plane wave gv is a non-flat solution of the field equations

$$R_{ij} = 0, \quad i, j = 1,2,3,4,5 \tag{2}$$

in an empty region of the space-time such that

$$ds^2 = g_{ij}\, dx^i\, dx^j$$

with $g_{ij} = g_{ij}(Z), \quad Z = Z(x^i), \quad x^i = u, x, y, z, t$ (3)

in some suitable co-ordinate system such that

$$g^{ij} Z_{,i} Z_{,j} = 0, \quad Z_{,i} = \frac{\partial Z}{\partial x^i}, \tag{4}$$

$$Z = Z(z,t) \text{ such that } Z_{,4} \neq 0,\ Z_{,5} \neq 0. \tag{5}$$

In this definition the signature convention adopted is

$$g_{aa} < 0, \quad \begin{vmatrix} g_{aa} & g_{ab} \\ g_{ba} & g_{bb} \end{vmatrix} > 0$$

[not summed for $a, b, c = 1,2,3,4$]

$$\begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix} < 0, \quad \begin{vmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{vmatrix} > 0, \quad g_{55} > 0 \tag{6}$$

and accordingly $g = \det(g_{ij}) > 0$ (7)

It is to be noted that, though the definition adopted by Adhav and Karade (1994) of plane wave is similar to that of Takeno (1961), the result (7) is contrary to that of Takeno (1961) which is obvious because of the signature difference. By reformulating Takeno's (1961) definition, Ladke et. al. and Zade et. al. obtain the plane wave solutions in six and four dimensional space- time respectively.

In this paper, I confine myself to the space-time used by Adhav and Karade (1994), but relax the conditions (3), (4) and (6) with assuming.

$$Z = Z(y, z, t),\ Z_{,3} \neq 0,\ Z_{,4} \neq 0,\ Z_{,5} \neq 0, \tag{8}$$

we get some interesting results and one can deduce the results of Adhav and Karade (1994) from ours which is generalization of their work.

[2] Plane Wave Solutions

From the equations (4) and (8) we get

$$g^{33}(Z_{,3})^2 + g^{34}Z_{,3}Z_{,4} + g^{35}Z_{,3}Z_{,5} + g^{43}Z_{,4}Z_{,3} + g^{44}(Z_{,4})^2 + g^{45}Z_{,4}Z_{,5}$$

$$+ g^{53}Z_{,5}Z_{,3} + g^{54}Z_{,5}Z_{,4} + g^{55}(Z_{,5})^2 = 0 \tag{9}$$

Dividing equation (9) by $(Z_{,5})^2$, we obtain

$$g^{33}\phi_1^2 + 2g^{34}\phi_1\phi_2 + 2g^{35}\phi_1 + 2g^{45}\phi_2 + g^{44}\phi_2^2 + g^{55} = 0 \tag{10}$$

$$\text{where } \phi_1 = \frac{Z_{,3}}{Z_{,5}},\quad \phi_2 = \frac{Z_{,4}}{Z_{,5}} \tag{11}$$

The solutions of (11) then yield

$$t + \phi_1 y = w_1, \tag{12}$$

$$t + \phi_2 z = w_2 \tag{13}$$

where w_1 and w_2 are arbitrary functions of Z.

Differentiating partially (12) with respect to *y, t* and (13) with respect to *z, t* , we obtain

$$Z_{,3} = \frac{\phi_1}{M_1},\quad Z_{,5} = \frac{1}{M_1} \tag{14}$$

Where $M_1 = \bar{w}_1 - \bar{\phi}_1 y$ (15)

and $$Z_{,4} = \frac{\phi_2}{M_2},\quad Z_{,5} = \frac{1}{M_2}. \tag{16}$$

Here $M_2 = \bar{w}_2 - \bar{\phi}_2 z$. (17)

Differentiating partially (15) with respect to *y, t* and (17) with respect to *z, t,* we get

$$M_{1},_{3}=\frac{N_{1}}{M_{1}}\phi_{1}-\bar{\phi}_{1}, \qquad M_{1},_{5}=\frac{N_{1}}{M_{1}} \tag{18}$$

with $N_{1}=\bar{w}_{1}-\bar{\phi}_{1}y$ (19)

and $$M_{2},_{4}=\frac{N_{2}}{M_{2}}\phi_{2}-\bar{\phi}_{2}, \quad M_{2},_{5}=\frac{N_{2}}{M_{2}}. \tag{20}$$

Here $N_{2}=\bar{\bar{w}}_{2}-\bar{\bar{\phi}}_{2}z$. (21)

From equations (14) and (16), (18) and (20), we obtain

$M_{1}=M_{2}=P$ (say) and $N_{1}=N_{2}=Q$ (say) (22)

Then the equations (14), (16), (18) and (20) can be rewritten as

$$Z,_{3}=\frac{\phi_{1}}{P}, \qquad Z,_{4}=\frac{\phi_{2}}{P}, \qquad Z,_{5}=\frac{1}{P}, \tag{23}$$

$$P,_{3}=\frac{Q}{P}\phi_{1}-\bar{\phi}_{1}, \quad P,_{4}=\frac{Q}{P}\phi_{2}-\bar{\phi}_{2}, \quad P,_{5}=\frac{Q}{P}. \tag{24}$$

It is to be noted that all above equations are in the format of Adhao and Karade (1994).

The Christoffel's symbols of second kind assume the values as under.

$$2P\Gamma^{i}_{ab}=-\bar{g}_{ab}w^{i} \tag{25}$$

$$2P\Gamma^{i}_{a3}=\phi_{1}g^{ij}\bar{g}_{aj}-\bar{g}_{a3}w^{i},$$

$$2P\Gamma^{i}_{a4}=\phi_{2}g^{ij}\bar{g}_{aj}-\bar{g}_{a4}w^{i},$$

$$2P\Gamma^{i}_{a5}=g^{ij}\bar{g}_{aj}-\bar{g}_{a5}w^{i},$$

$$2P\Gamma^{i}_{33}=2\phi_{1}g^{ij}\bar{g}_{3j}-\bar{g}_{33}w^{i},$$

$$2P\Gamma^{i}_{34}=[g^{ij}(\phi_{2}\bar{g}_{3j}+\phi_{1}\bar{g}_{4j})-\bar{g}_{34}w^{i}],$$

$$2P\Gamma^{i}_{35}=[g^{ij}(\bar{g}_{3j}+\phi_{1}\bar{g}_{5j})-\bar{g}_{35}w^{i}],$$

$$2P\Gamma^i_{44} = 2\phi_2 g^{ij}\bar{g}_{4j} - \bar{g}_{44}w^i,$$

$$2P\Gamma^i_{45} = [g^{ij}(\bar{g}_{4j} + \phi_2\bar{g}_{5j}) - \bar{g}_{45}w^i],$$

$$2P\Gamma^i_{55} = 2g^{ij}\bar{g}_{5j} - \bar{g}_{55}w^i,$$

Here $w^i = \phi_1 g^{3i} + \phi_2 g^{4i} + g^{5i}.$ (26)

Noting w^i, equation (11) yields

$$\phi_1 w^3 + \phi_2 w^4 + w^5 = 0 \qquad (27)$$

Using (25), (26) and (27), the field equations $R_{ij} = 0$ then yield

$$\rho_a = \bar{g}_{ai}w^i = 0 \qquad (28)$$

and $Q\rho_{\alpha\beta} + P\sigma_{\alpha\beta} = 0$ (29)

$R a\alpha = 0$ where $a = 1,2$ and $\alpha,\beta = 3,4,5$ (30)

where $R_{ab} = \dfrac{\rho_a\rho_b}{2P^2}$,

$$\sigma_{33} = -\bar{\rho}_{33} + \frac{1}{4}\left[\phi_1^2 L_1 - 4\phi_1 L_2\rho_3 + 2\rho_3^2\right],$$

$$\rho_{33} = -\phi_1^2 L_2 + \phi_1\rho_3,$$

$$\sigma_{34} = -\bar{\rho}_{34} + \frac{1}{4}\left[\phi_1\phi_2 L_1 - 2L_2(\phi_2\rho_3 + \phi_1\rho_4) + 2\rho_3\rho_4\right],$$

$$\rho_{34} = \frac{1}{2}\left[\phi_2\rho_3 + \phi_1\rho_4\right] - L_2\phi_1\phi_2,$$

$$\sigma_{35} = -\bar{\rho}_{35} + \frac{1}{4}\left[\phi_1 L_1 + 2\rho_3\rho_5 - 2L_2(\rho_3 + \phi_1\rho_5)\right],$$

$$\rho_{35} = \frac{1}{2}\left[\rho_3 + \phi_1\rho_5\right] - L_2\phi_1,$$

$$\sigma_{44} = -\bar{\rho}_{44} + \frac{1}{4}\left[\phi_2^2 L_1 + 2\rho_4^2 - 4L_2\phi_2\rho_4\right],$$

$$\rho_{44} = \phi_2\rho_4 - \phi_2^2 L_2,$$

$$\sigma_{45} = -\bar{\rho}_{45} + \frac{1}{4}\left[\phi_2 L_1 + 2\rho_4\rho_5 - 2L_2(\rho_4 + \phi_2\rho_5)\right],$$

$$\rho_{45} = \frac{1}{2}\left[\rho_4 + \phi_2\rho_5\right] - L_2\phi_2 ,$$

$$\sigma_{55} = -\bar{\rho}_{55} + \frac{1}{4}\left[L_1 - 4L_2\rho_5 + 2\rho_5^2\right],$$

$$\rho_{55} = \rho_5 - L_2 ,$$

and $$\rho_i = \bar{g}_{ij} w^i, \quad L_2 = \overline{\log\sqrt{g}}, \quad L_1 = g^{ij} g^{kl} \bar{g}_{ik} \bar{g}_{jl} . \qquad (31)$$

Substituting the values of Q and P in equations (2.2.28) we get

$$\bar{\bar{w}}_1 \rho_{\alpha\beta} + \bar{w}_1 \sigma_{\alpha\beta} = 0 = \bar{\bar{\phi}}_1 \rho_{\alpha\beta} + \bar{\phi}_1 \sigma_{\alpha\beta} ,$$

$$\bar{\bar{w}}_2 \rho_{\alpha\beta} + \bar{w}_2 \sigma_{\alpha\beta} = 0 = \bar{\bar{\phi}}_2 \rho_{\alpha\beta} + \bar{\phi}_2 \sigma_{\alpha\beta} \qquad (32)$$

All these equations follows the pattern of Adhav and Karade (1994).

3] CONCLUSION

We conclude that the plane wave solutions of $R_{ij}=0$ are given by g_{ij} which satisfy (3), (6), (27), (28) and (32). If the function Z is independent of the variable y then the work of Adhav and Karade (1994) regarding plane wave solutions in five dimensional space-time V_5 can be brought out.

REFERENCES

1.Takeno H (1961) ' The Mathematical Theory of Plane gravitational waves in general relativity', Scientific report of the research institute for theoretical Physics, Hiroshima university, Takchara, Hiroshima-ken, Japan.

2.Adhav K S and Karade T M (!994) Post Raag Reports, Japan, No- 279

3.Ladke L. S. and Thengane K. D. (2001) Bulletin of pure and applied sciences Vol. 20E (No. 2) p 403-408.

4. Zade V. T., Thengane K. D. And karade T. M. , (2002) Mathematical Education, Vol. XXXVII, No. 3

L. S. Ladke
Department of Marhematics , Sarvodaya Mahavidyalaya Sindewahi Distt. Chandrapur India.