
REAL TIME MODELLING AND BALANCE CONTROLLER DESIGN FOR A ROTARY INVERTED PENDULUM – USING LabVIEW

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Abstract

The Quanser Rotary Inverted Pendulum is highly non-linear, open loop and unstable system. Solving the operation of the Quanser rotary inverted pendulum is one of the classical and fundamental problems in the area of control theory. The practical controller design and implementation for such a system is a challenging task. The main objective is to design a stabilizing controller that balances the inverted pendulum in the up-right position. This paper describes modern control technique that include Full State Feedback (FSF) controller design to control the Rotary Inverted Pendulum using LabVIEW Interface.

KEY WORDS:

FSF (Full state-feedback), LabVIEW (Laboratory Virtual Instrumentation Engineering Workbench), Quanser Rotary Inverted Pendulum, Stabilizing Controller.

INTRODUCTION

There is a lot of research that has been done on the of quanser rotary inverted pendulum. This is good topic for application of different control strategies. The principle of its stabilization can be found at many devices. Most famous equipment is Segaway. It is self - balancing robotic mobility platform for personal transport. Further applications of the stabilization inverted pendulum principle have been use in the literature. In paper [1] Kumagai and Ochiai designed a robot which balanced on a ball. Robot could be realized using two mechanisms, one was inverted pendulum control in two directions and other was an omnidirectional mechanism for driving the ball. Gaiceanu and Stan used this stabilization system to motion control of a Single-beam gantry crane trolley [2]. There are many other facilities where it is possible to apply principle of the inverted pendulum stabilization.

The rotary inverted pendulum is highly nonlinear and open-loop unstable system. The controller design for such a system is a challenging task. The most commonly use control system is PID controller. It consists of three separate parameters, proportional, integral and derivative. Proposal of these parameters to achieve the desired behavior as settling time, overshoot and steady state error is not easy. Although it can be done using Simulink Control Design PID Tuner to tune PID gains automatically in a Simulink model. To design of feedback controller, there are also other techniques such as Niquist and Bode plots or root locus plots.

Another alternative is the use of state-feedback control system also known as pole placement. Placing poles is desirable because the location of the poles determines the eigenvalues of the system, which controls the characteristics of the system response. State-feedback algorithm is actually an automated technique to find an appropriate state-feedback controller [3].

In this paper we study stabilization problem of the rotary inverted pendulum and the design of

control system using pole placement technique is presented. Experiments have been done in LabVIEW environment and in real-time experiments on the rotary inverted pendulum system.

II. ROTARY INVERTED PENDULUM MODEL

(1) Mathematical model

A mathematical model is the set of equations which describe the behavior of the system. A state space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. The state space representation provides a convenient and compact way to model and analyze systems with multiple inputs and outputs. The following assumptions are important in modeling of the system:

- 1) The system starts in a state of equilibrium meaning that the initial conditions are therefore assumed to be zero.
- 2) The pendulum does not move more than a few degrees away from the vertical to satisfy a linear model.
- 3) A small disturbance can be applied on the pendulum.

(2) Modelling

Figure 1 depicts the free body diagram of pendulum from which equations of motion can be derived.

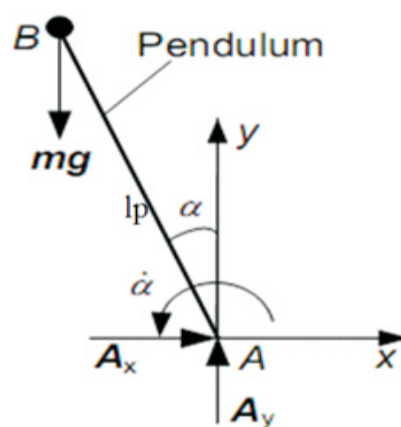


Figure 1:Free body diagram of pendulum

Using torque balance equations;

The two components for velocity of the pendulum center of mass are;

$$V_{pend.center\ of\ mass} = l_p \cos(\dot{\alpha})\dot{\alpha}\hat{x} + l_p \sin(\dot{\alpha})\dot{\alpha}\hat{y} \tag{1}$$

The pendulum arm moves with rotating arm at a rate of;

$$V_{arm} = r\dot{\alpha} \tag{2}$$

The x and y velocity components are;

$$V_x = r\dot{\alpha} \cos(\alpha) \tag{3}$$

$$V_y = l_p \sin(\alpha)\dot{\alpha} \tag{4}$$

The dynamic equations can be obtained using Euler-Lagrange equations;

a)Potential energy:

Gravity is the only potential energy in the system. Potential energy is given by equation A(5)

So,

$$V = P.E_{pend} = M_p g l_p \cos \alpha \tag{5}$$

b)Kinetic energy:

Complete kinetic energy of the system is given by equation A(6)

$$T = K.E_{arm} + K.E_{pend} + K.E_{V_x} + K.E_{V_y} \tag{6}$$

Lagrangian of the system is given by equation (7),

$$L = T - V \tag{7}$$

$$T = \frac{1}{2} J_{eq} \dot{\alpha}^2 + \frac{1}{2} J_p \dot{\theta}^2 + \frac{1}{2} M_p r^2 \dot{\alpha}^2 + \frac{1}{2} M_p l_p^2 \dot{\theta}^2 + M_p r \dot{\alpha} l_p \cos \alpha \dot{\theta} \tag{8}$$

Euler–Lagrange equations of motion is given by equations (9) and (10),

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\alpha}} \right) - \frac{\partial L}{\partial \alpha} = B_{eq} \dot{\theta} \tag{9}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = B_p \dot{\alpha} \tag{10}$$

Applying equations (9) and (10) in equation (8) and linearizing about $\alpha=0$, we get

$$J_{eq} \ddot{\alpha} + M_p r^2 \ddot{\alpha} + M_p l_p r \ddot{\theta} = B_{eq} \dot{\theta} \tag{11}$$

$$J_p \ddot{\theta} + M_p l_p^2 \ddot{\theta} + M_p l_p r \ddot{\alpha} + M_p g l_p \theta = B_p \dot{\alpha} \tag{12}$$

Torque generated at arm pivot by

$$output = \frac{K_t V_m - K_t K_m \dot{\theta}}{R_m} \tag{13}$$

State variables,

$$x_1 = \alpha, x_2 = \dot{\alpha}, x_3 = \theta, x_4 = \dot{\theta} \tag{14}$$

B_{eq} and B_p terms are regarded as negligible

Sub equations (13) and (14) in equations (1) and (2), we get

$$(J_{eq} + M_p r^2) \dot{x}_3 + M_p l_p r \dot{x}_4 = \frac{K_t V_m - K_t K_m \dot{\theta}}{R_m} \tag{15}$$

$$(J_p + M_p l_p^2) \dot{x}_4 + M_p l_p r \dot{x}_3 + M_p g l_p x_3 = 0 \tag{16}$$

By solving equations (15) and (16) , the following state space representation of the complete system is obtained, State equation is given by equation (17),

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & bd/E & cG/E & 0 & cK_t/R_m E \\ 0 & ad/E & bG/E & 0 & bK_t/R_m E \end{bmatrix} V_m \quad (17)$$

Output equation is given by equation(18),

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} V_m \quad (18)$$

Where,

$$a = J_{eq} / M_p r^2, b = M_p l_p r, c = (J_p + M_p l_p^2), d = M_p g l_p, E = ac - b^2$$

$$G = \frac{K_t K_m}{R_m}$$

The values are then used in the state space model to obtain the numerical representation of the system for the purpose to design the controller.

III. FSF CONTROLLER DESIGN

The system of the inverse pendulum is unstable. With help of pole placement method it is possible to make stable existing system[4]. Eigen values that are not situated in the unit circle have to be moved to the desired position into this unit circle. Input vector should be designed using the state feedback control law:

$$u(k) = -K.x(k) \quad (19)$$

We can place closed-loop poles at will use the following full-state feedback gain matrix given by:

$$K = [K_1, K_2, K_3, K_4] \quad (20)$$

Since the system is fourth order, each pair of closed loop poles is introduced into the left semi plane of the complex plane.

The closed loop system matrix (A - B L) has eigenvalues that coincide with the zeros of the specified desired characteristic polynomial . Poles of the closed loop system are given by:

$$|zI - (A - BK)| \quad (21)$$

Ackermann's formula is used for inverted pendulum control, with a view to determine full-state feedback gain (20). It is generalized in the sense of direct us of desired eigenvalues instead of characteristic polynomial coefficients.

Using of state space feedback can affect the internal state of rotary inverted pendulum system. Fig. 2 illustrates block diagram of a FSF Controller

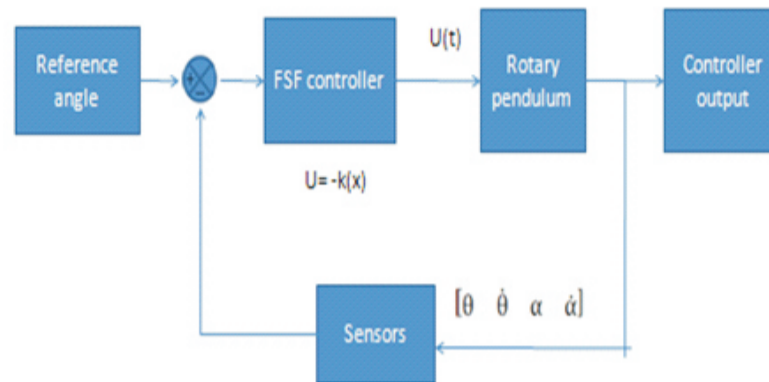


Fig (2) Block diagram of FSF controller

IV. SIMULATION RESULTS

The experimental results from simulation model in LabVIEW and measured results on the real sample are shown on figures below.

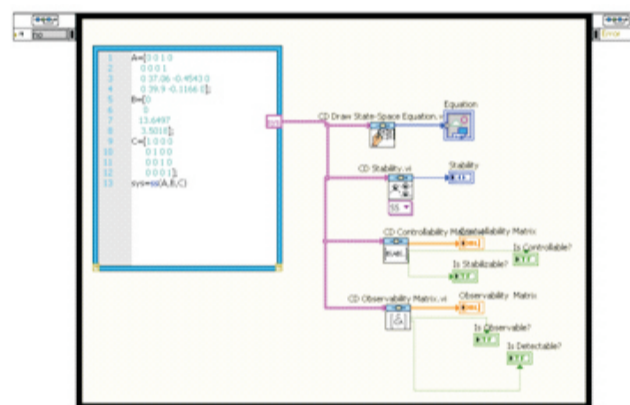


Fig (3) Block diagram of controller

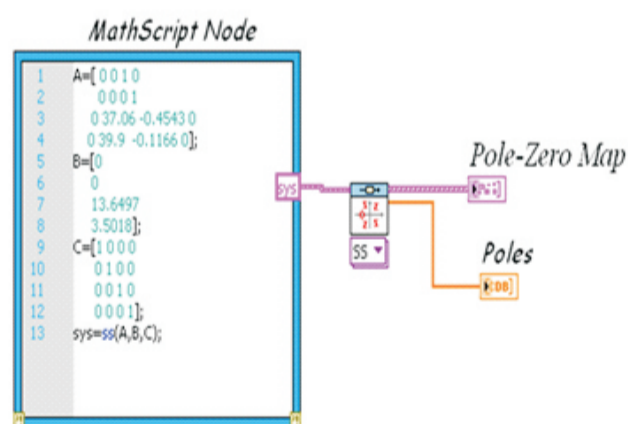


Fig (4) Block diagram of a pole-zero block

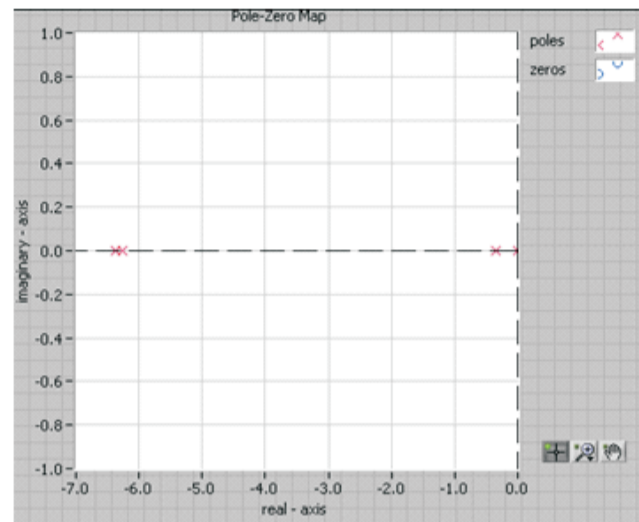


Fig (5) pole zero mapping

$$\text{Equation}$$

$$\frac{dx}{dt} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0.00454273 & -291.706 & 0.469334 & -52.4843 \\ 0.00116543 & -44.4441 & 0.120356 & -13.4647 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 13.6497 \\ 3.5018 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

Fig (6) state space equation from front panel

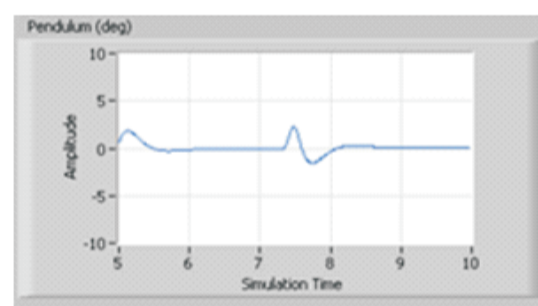


Fig (7) Response of the pendulum after Pole placement

V. CONCLUSION

In this paper has been presented the application of state feedback control for regulation & stabilization of inverted pendulum. Here, pole-placement method is used to put desired poles in our target position. So following system is stabilized and controllable by using full state-feedback method. Pole placement technique is insensitive to system parameter variations and external disturbances. It remains stable even in the case of model approximation errors. The simulation results demonstrated that the system can successfully be handled by using FSF controller.

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